Computer Assisted Composition in Equal Tunings, and the *Thirteen Tone March*

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1 Abstract

This essay first describes the technological method I used to create a working environment for creating music with equal tunings in divisions of the octave other than 12. It then makes some general comments on the tuning of thirteen equal tones, and presents a partial analysis of my own composition in this tuning. This composition, simply entitled "Thirteen Tone March" was created as part of a project to attempt the composition of tonal music in tuning systems with numbers of equal tones higher than twelve. The essay concludes with an aesthetic discussion of the reasons for such a project, and its results.

2 A Working Environment for Composing in Equal Tuning

In standard equal tuning, the octave is divided into twelve equal steps. Given any frequency F, one can determine the frequency of another note x that is y of these steps away by the following formula:

$$f(x) = F \cdot 2^{\frac{y}{12}} \tag{1}$$

Intuitively enough, the resulting note is y steps above the known frequency F if y is positive, and y steps below F if y is negative.

To divide the octave into equal steps numbering other than 12, we can replace 12 with a variable, d (standing for 'division' of the octave).

To implement this in a MIDI interface,

- Let M be the MIDI note number that corresponds to the known frequency F
- Let x be the MIDI note number of a key on a controller that has been played
- The distance y can now be understood as the difference in MIDI note numbers of the played and known notes, or $x - M$. (This order preserves the positive-negative relationship described above.

Thus, given a known note M with frequency F, the frequency of any note x can be expressed as below:

$$f(x) = F \cdot 2^{\frac{x-M}{d}} \tag{2}$$

Using the formula above, I created a retuning patch in the Max/MSP environment that lets the user assign values to d, F , and M ; it then accepts MIDI input and outputs frequency in Hz (and passes velocity through unchanged). Its output can be sent to any synthesis module that accepts frequency and velocity. To ensure problem-free polyphony, a simple but safely redundant mechanism is put into place.

As an example, the common reference pitch A 440 would let M equal 69 (on some keyboard controllers) and F be 440.000 Hz. Playing a note on a MIDI controller 12 physical keys above the given A would let x equal 81, $x - M$ would be 12, and if $d = 12$ then $f(x)$ would be 880.000 Hz, one octave above the reference note. If d were 13, however, one would need to play a note 13 physical keys higher then the reference note to produce an octave. It would look like an A-sharp on the keyboard, but would sound an A. Similarly, playing a C key three physical keys above the reference A while $d = 12$ would obviously result in a C (523.251 Hz) or 3/12 of an octave. While $d = 13$, playing the same key would sound only 3/13 of an octave above A (516.323 Hz), while $d = 18$ it would sound 3/18 of an octave above, and so on.

The physical key corresponding to the reference note M will be the one key that is never retuned, and all of the other keys on the controller will be retuned around it. For this reason, I refer to the reference note as the pole. F and M can be set together by picking a pole; in this case it is assumed that this note will have the same frequency that it does in standard tuning. In my scores written for this environment, the tuning is indicated at the top of the page by giving values for d and for the pole.

3 Tuning in Thirteen Equal Tones

I greatly enjoy Easley Blackwood's microtonal compositions and am impressed with his essay from around fifteen years ago that proposes some useful theory for dealing with nineteen, seventeen, sixteen and fifteen note equal tunings. His approach to these tuning systems is well adapted to the peculiarities of each system. He uses the nineteen and seventeen note equal tunings as systems in which diatonic scales can be made, with the relationship of the 'half step' interval to the 'whole step' interval set to ratios other than 1:2 (2:3 in the case of nineteen tones, and 1:3 in the case of seventeen). The sixteen note equal tuning is approached as a similar variation on what we call an "octatonic" scale in our usual system of twelve equal tones. Finally, the fifteen note equal tuning is given an intricate treatment in which interest arises from an interplay between ten- and a six-note symmetrical modes. My approach to the tuning of thirteen equal tones per octave is to take advantage of how close some intervals sound to our usual twelve note tuning. In many cases I speak of "approximations of" or "attempts at" elements of the standard twelve note tuning. After working in this tuning for some time, I created the Thirteen Tone March, each section of which showcases a different interesting aspect of the thirteen note equal tuning. As for notation, I simply add an extra, thirteenth note to our usual twelve note system. This note is C-flat, written with a backwards flat symbol as a visual reminder that this note is different than usual and is no longer enharmonic with B-natural. It would, in theory, be enharmonic with a B-sharp (written with some form of modified sharp symbol), but no use for this was found in the March. It is possible to use the Max/MSP patch described above as a tool for performing live in alternate equal tunings as well. When this is done, I prepare an "action-based score" that corresponds to the physical keys on the keyboard that the keyboardist will play, instead of the resulting sound. However, the sound-based score is more convenient for the analytical purposes of this paper, and is included at the end of this document. A recording is available online.[1]

4 The First Strain: Approximations of the Major Triad

The first strain (mm. 5-21) features an interplay between two approximations of the major tonic triad. From the tonic, F, we find two major thirds that are close to the 12-tone major third to which we are accustomed. A-sharp will be a little higher than normal, and A will be a little lower than normal (but not so low as to be heard as a minor third). Likewise, there are two candidates for a perfect fifth above tonic: C is a little higher than a perfect fifth above F, and the new note, C-flat, is a little lower. Triads that sound somewhat close to the usual major triad can be built by using the higher third with the higher fifth, or by using the lower third with the lower fifth. It is convenient to think of these two approximations of the major triad as the tall and short major chords. Mixing the low third with the high fifth (F, A, C) sounds closer to a C major chord in inversion, and mixing the high third with the low fifth creates a chord that doesnt sound like a triad at all. Fig. 1 shows the acceptable tall and short versions of the tonic F triad.

Figure 1: Tall and Short F Major Chords

Use of these two different tonic triads can be seen at the beginning of the first strain. The introduction (mm. 1-4) had presented the short tonic chord. The first strain begins

[1]http://www.JohnChowSeymour.com/13march.html

with the tall tonic chord in measure 5 moving to a dominant chord (to be discussed later) in measure 6, which resolves in measure 7 to the short tonic chord.

A more melodic example occurs in measure 11. To finish the sequence set up in measures 8-10, the melody in measure 11 could easily have been C, A-sharp, F the tall major chord. Instead of the F, however, the fifth and third of the short major chord are heard, and the tonic F is finally heard in measure 12.

In both of these examples, the short major chord is treated as the correct one, at least in that it gets the final word. An example of the opposite order can be seen in measure 19. This time, however, the gesture takes place on the level of the dominant, so a look must be taken at the dominant chord(s) first.

Because there are two approximations of the dominant note (C and C-flat in this case), and two approximations of any major chord, there are four total possibilities for a dominant chord. All four possibilities are shown in Fig. 2.

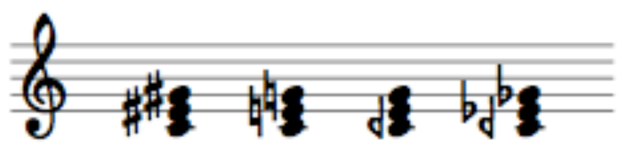

Figure 2: Dominants: Tall and short major chords built from C and from C-flat.

In measure 19, a similar gesture to the one in measure 11 is seen, on the level of the dominant. Here, the fifth of the short major chord, C-flat, is used as the dominant note. First the short major chord over C-flat is heard (G-flat then E-flat melodically, with C-flat in the bass), then the tall one (G, E, C-flat melodically; the C-flat is on the downbeat of measure 20).

A larger harmonic gesture is also at work here. The first strain is, in standard March form, expected to end on the dominant, but which one? In measure 17, a dominant chord is built on the tall dominant note, C. For melodic reasons, it happens to be the short chord (C, E, G). Next, the familiar (if reversed) gesture discussed above is heard over the short dominant note, C-flat: first the short chord over this note, then the tall, as discussed. Although the phrase ends on a tall dominant chord, it is the short dominant note that is confirmed, just as the same note, C-flat, was preferred at the endings of phrases previously in the first strain as the fifth of the short tonic chord. The next section of this paper proposes a reason for using the tall chord here.

5 Two Major Modes with the Same Tonic

It is possible to theorize modes from which the tall and short tonic chords are built. Moving downward from F in the pattern of the descending major mode (half, whole, whole etc.) each melodic whole step is only a little flatter than it would be in a system of twelve equal tones (2/13 of one of the twelve-note system's half steps), and each thirteen-note half step is even less different (1/13 of a twelve-note half step). Of course, these fractions add up the more steps one takes, but this additive effect is not as noticeable as the result of stepwise motion as it is when larger intervals are played melodically. Unfortunately, after seven steps of this descending scale, we arrive not at the tonic F but at an F-sharp. (we are, after all, now 12/13 of a half step off: inverted, this is one entire half-step of the thirteen note tuning).

Likewise, starting at the same tonic and moving upward in the usual whole-whole-half etc. diatonic pattern of a major scale, a different major scale is attempted, with each note a written half-step lower than the corresponding scale degree in the scale built by descending from the tonic. Where an octave should be, after seven steps, we find F-flat instead. These two failed attempts at complete major scales are shown in Fig. 3.

Figure 3: Two attempted major scales with tonic F.

It can then be theorized that the tall tonic triad (F, A-sharp, C) is constructed with the tonic F (common to both scales) plus the third and fifth from the higher scale above. The short tonic triad (F, A, C-flat) is made with the tonic plus the third and fifth from the lower of the above scales. With this in mind, it could also be said that many of the interesting features of the March can be thought of as interplay between these two scales.

Notice that none of the four dominant chords can be spelled entirely with notes from one scale or the other; any dominant chord requires using notes from both of them. Note also that only two of the chords, the two that have E-G as their upper third, contain their third as scale degree seven and their fifth as scale degree two. The tall chord over the tall dominant contains E-sharp as its third, and this note is only found in either scale enharmonically as F, the tonic. Similarly, the short chord over the short dominant note contains G-flat, which is only found enharmonically as F-sharp, the incorrect tonic of the descending scale. This is why I chose to end the first strain on the tall chord built on C-flat: the short chord over C-flat is less relatable to the scale system suggested above and thus seemed more of a foreign element than a true dominant.

6 The Second Strain: Chromatic Lines, and a Half-Step Tonicization

In the second strain (mm. 22-38), I decided to feature small, often chromatic melodic intervals, to demonstrate how natural and inoffensive chromatic lines sound in 13-tone tuning.

The other feature of the second strain may be more interesting analytically, however. Both the tall and short major chords use the same interval between their third and fifth, a minor third. Because this is so, the upper notes of a tall tonic triad could also be the upper notes of a short major chord built on the note a half step above tonic. Fig. 4 shows this relationship.

Figure 4: Tall F Major and Short F-Sharp Major chords.

While tonicizing a chord one half-step above tonic can be awkward in twelve tones, it can be very smooth in thirteen. An example of such a tonicization occurs in measures 28-29. The method is simple, the descending scale of Fig. 3 is allowed to continue melodically to its incorrect tonic, F-sharp. The bass line ascends to the short fifth of the new F-sharp chord, and the two voices sound the short third of the F-sharp chord at the same time.

Feeling the need for more tension at the end of the second strain, I decided to try reversing the order of tall and short tonic chords. In measure 34, a short tonic chord is heard. As the bass alternates between the tall and short dominant notes, both of the viable dominant chords are heard in measure 35, ending with the one that includes the tall dominant note, C. The short tonic in measure 36 seems to end the phrase, but then the rhythm continues with two statements of the tall tonic. After so much favoring of the short tonic, the tall tonic here sounds unusual, especially since its third and fifth were used earlier to tonicize a different root, F-sharp.

7 The Trio: Large Melodic Intervals

The third section of the standard March form, called a "Trio," is usually said to be in the key of the subdominant. In my march, I decided once again to favor the lower of the two scales, and use B-flat as the subdominant. In contrast to the chromatic motif of the second strain, the trio features the wider intervals that sound more noticeably out-of-tune in the 13-tone system. To increase energy, the melodic intervals tend to get bigger as the section moves on. Note the descending scale in measures 67-68. It starts out as the descending scale from Fig. 3, for four notes, and switches to the bottom part of the ascending scale in Fig. 3 (here descending), for the remaining five notes. Both the tall and short dominant notes (F, and F-flat = E) are heard, here presented melodically with a half step between them. Finally, the first ending of the Trio (measure 70) presents the short tonic chord (B-flat, D-flat, F-flat), but the second ending presents the tall tonic chord (B-flat, D, F). This is a reversal of the usual finality of the shorter major chord. My reasons for doing so are this time not theoretical but psychological, and will be discussed in the next, final section of this paper.

8 Tonality Without the Twelve-Note System

One of the goals for this project was to explore tonality via parody; the parody being not only of the standard military march form but also in that the 13 tone tuning was used to create a parody of traditional tonality. The tonal aspects of this form are recognizable even to someone untrained in their analysis, and yet, the system of twelve tones by which we often define tonality is missing or at best approximated. What does it say about our relationship to tonality that we can hear it when it isnt technically there?

One obvious theory about this would be that a listener acculturated in twelve-note tonality, if not specifically military marches, would unconsciously try to find familiar tonal structures in this music. Related to this is the idea of "tolerance," invoked by many music analysts in order to dismiss pitch inaccuracies during a performance or differences in historical tuning systems as non-fatal to an analysis in 12 theoretically equal tones. The idea is that our ears, acculturated to music in 12 tones, will ignore deviations from theoretically exact intervals within a given tolerance range. In tonal music in 13 tones, there are often two notes than can approximate any one note of the usual 12-note system, as demonstrated earlier; and the composer can utilize this ambiguity to confuse the listeners usual subconscious mechanism that ignores pitch deviations, and to challenge our ability to "tolerate" deviations and still hear this music as tonal.

With this in mind, there were times in this composition at which I took measure to keep the listener from becoming too comfortable with the new tuning system to heighten the sense of parody and humor, but also to ex- periment with our subconscious tendencies toward tonal listening. In an early version of the first strain in which I had used only one dominant note in the bassline (C, and never C-flat) my ears found it easy to simply adjust to the extra-wide perfect fifth. In the final version, however, C and C-flat interchange in unpredictable ways. While the C-flat probably sounds like a "wrong note" in the bassline of measure 8 (after so many bassline gestures featuring only C and F), it sounds more correct in measure 11 (now supported by the melody note) and more or less confirmed in measure 20 (approached with a tall dominant note of its own, G). In this and other ways, I took measure to make sure that the listener does not simply grow accustomed to a slightly out-of-tune version of twelve-note tonality: I made the most of the ambiguities of the thirteen-note system. And yet, the tonal moments in the piece are unmistakable.

Theorists often speak of music in terms of expectation; its dramatic power is said to come from the denial and eventual fulfillment of what we expect to hear. In a traditional tonal context, we expect to hear the tonic at the end of the piece. As mentioned earlier, I ended the piece on the tall tonic instead of the usual preference in which the short chord gets the "last word." Since it is the tonic, but not the the tonic we were expecting, are our expectations adequately fulfilled? It may be that, despite the tall tonics unexpected arrival, we find it close enough to our tonal expectations to be heard as a tonic chord. After all, while I wrote often of "approximate" chords and "attempted" scales, the note F in the first two sections of this piece is made to be the tonal center in a way that is not merely approximate, despite my efforts to undermine any sense of a consistent dominant note or tonic chord.

References

[1] Blackwood, Easley. Modes and Chord Progressions in Equal Tunings, *Perspectives of New Music* Vol. 29 No. 2 (Summer 1991):166- 200